15 Years of Instant Radiosity

Alexander Keller

NVIDIA

Outline

Light Transport Simulation

- Using many point light sources
- The accumulation buffer
- Light path vertices as point light sources
- Instant radiosity

Path Space Partitioning

- The weak singularity

Consistent Generation of Light Transport Paths

- Monte Carlo and quasi-Monte Carlo integration

15 Years of Instant Radiosity

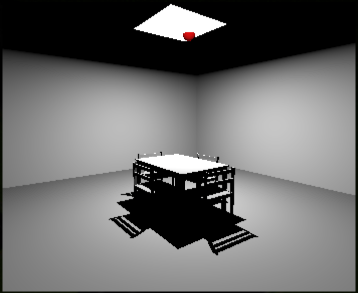
- Hindsights

Light Transport Simulation

Using many point light sources

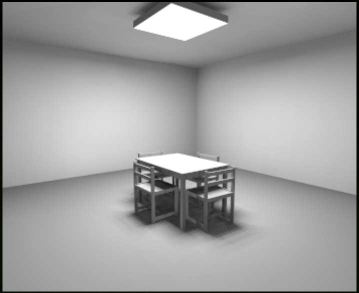
- accumulation of images illuminated by a point light source

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1}$$



i

$$=$$

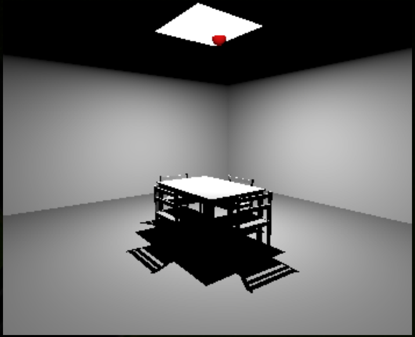


rendered by the rasterizer with shadow maps

Light Transport Simulation

The accumulation buffer

- example: approximate area light source by point light sources



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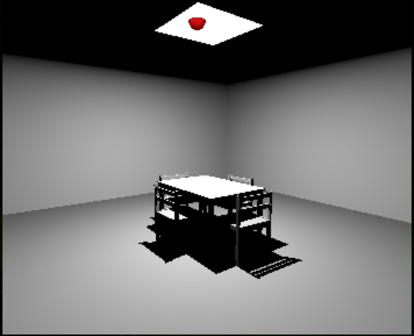

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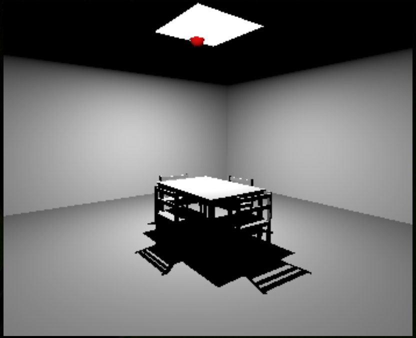

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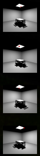

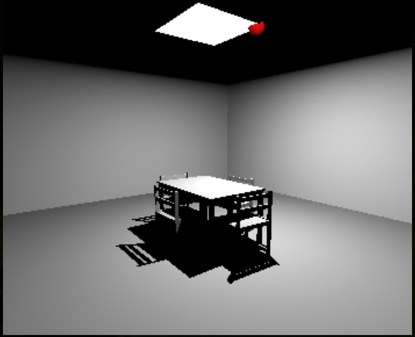

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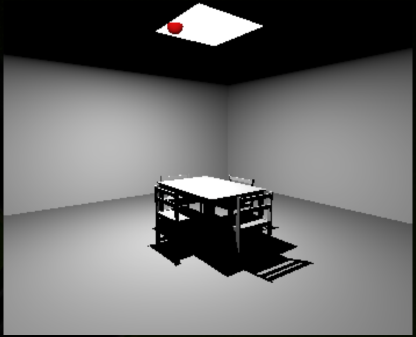

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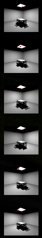

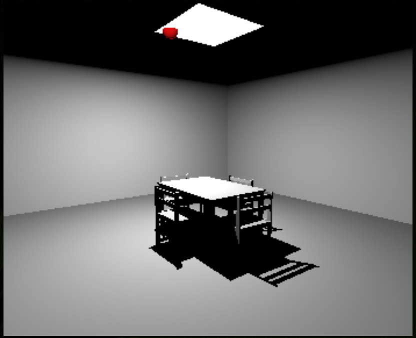

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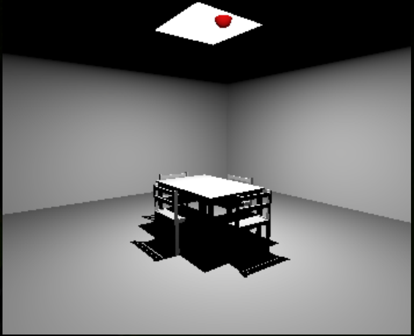

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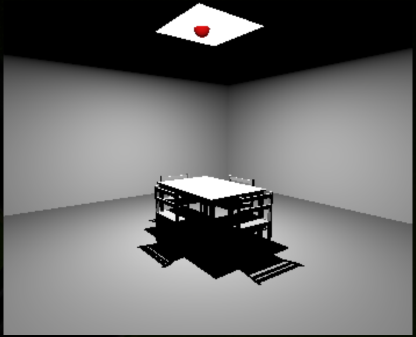

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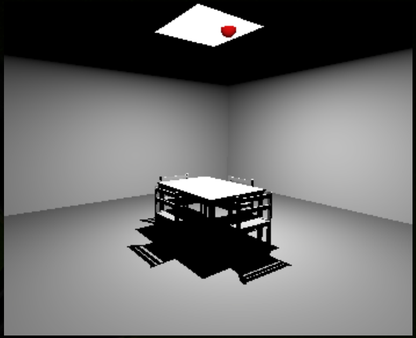

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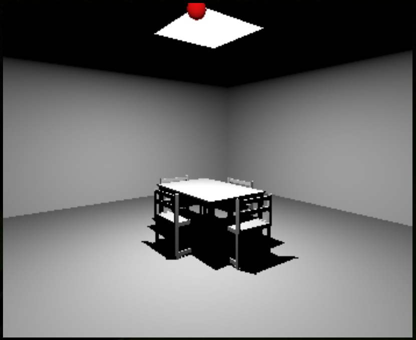

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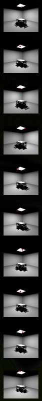
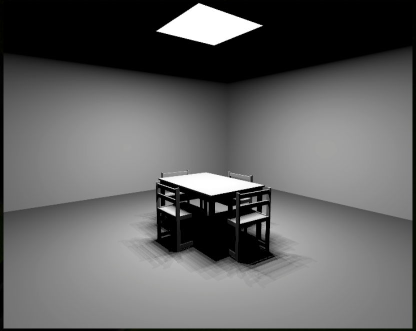
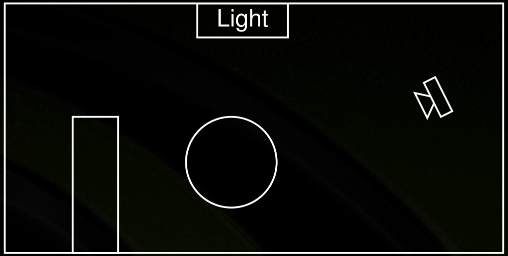




Light Transport Simulation

Light path vertices as point light sources

- trace light paths and store vertices $(x_j, L_j(x_j \to \cdot))_{j=1}^{M}$ as point lights



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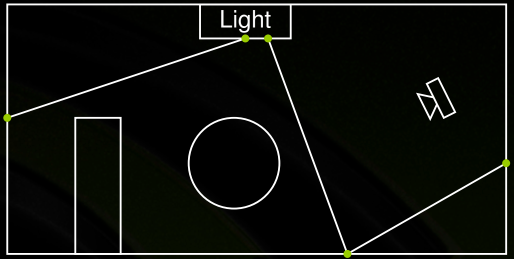
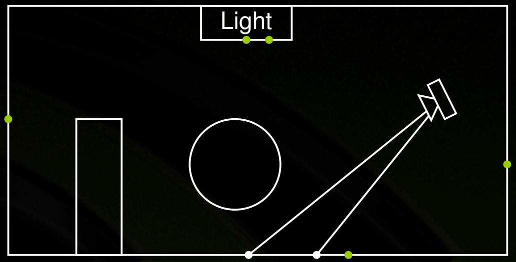


in fact equivalent to generating a tiny photon map

Light Transport Simulation

Light path vertices as point light sources

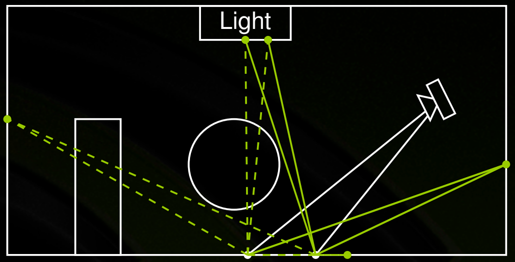
- trace camera paths



Light Transport Simulation

Light path vertices as point light sources

- trace shadow rays to illuminate camera paths by points lights



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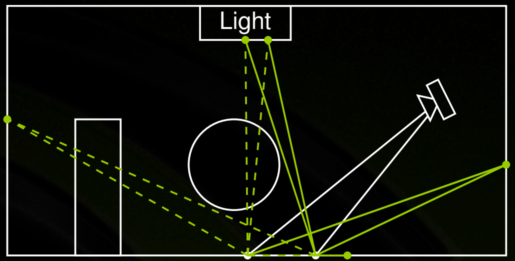
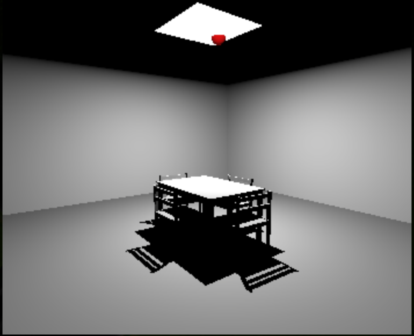


$$L(y,z) \approx L_e(y,z) + \sum_{j=1}^{M} L_j(x_j \to y) f_r(x_j,y,z) V(x_j,y) \frac{\cos\theta_{x_j} \cos\theta_y}{|x_j - y|^2}$$

Light Transport Simulation

Instant radiosity

- accumulation of images illuminated by a point light source



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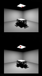

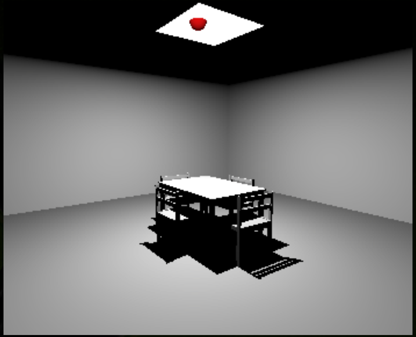

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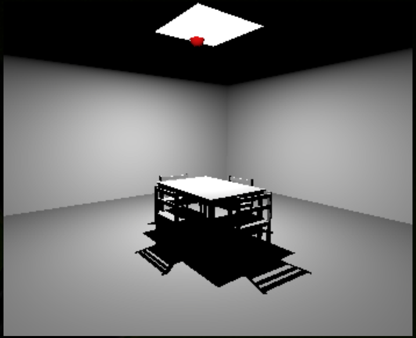

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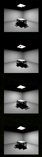

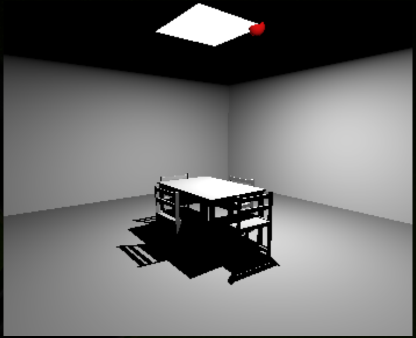

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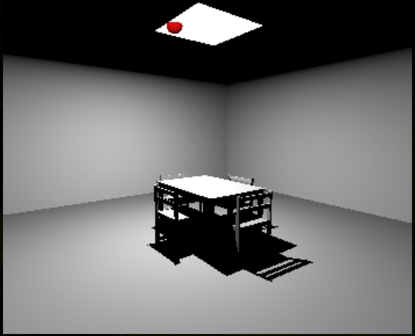

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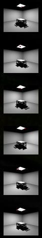

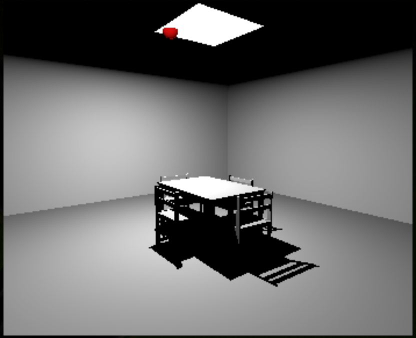

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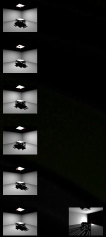

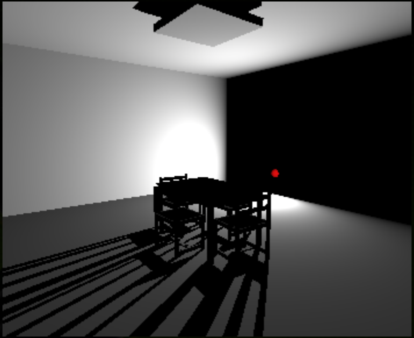

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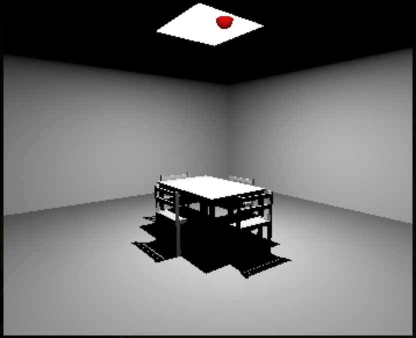

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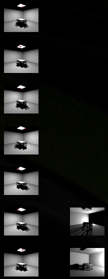

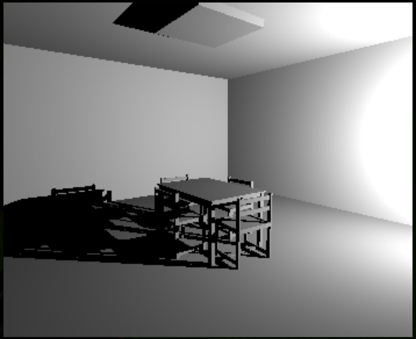

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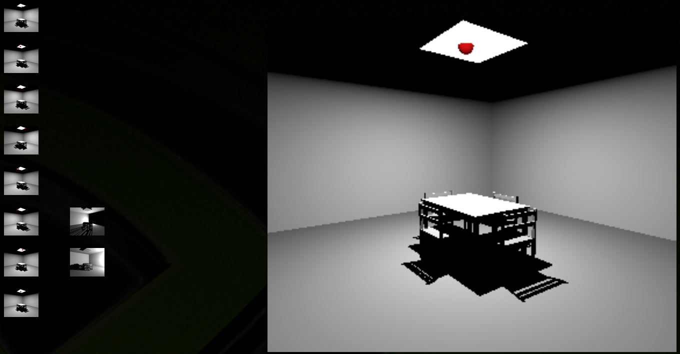

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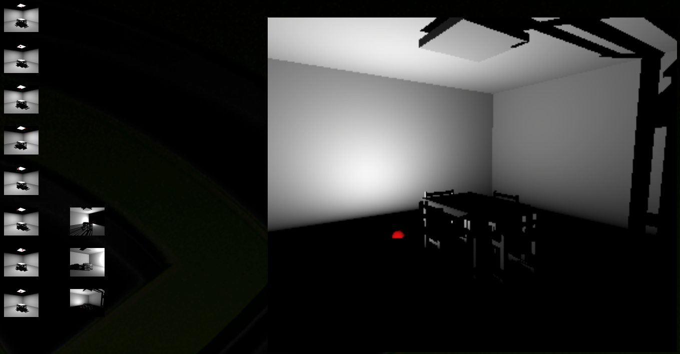

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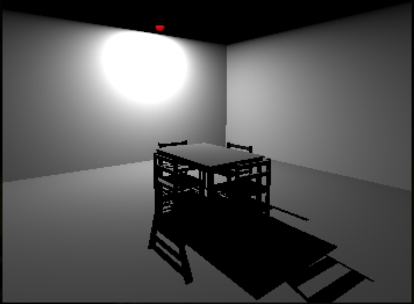

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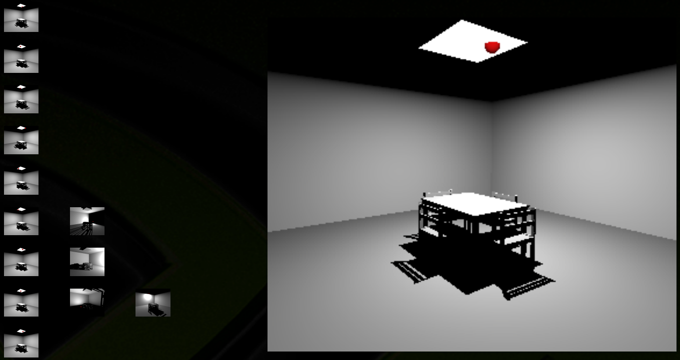

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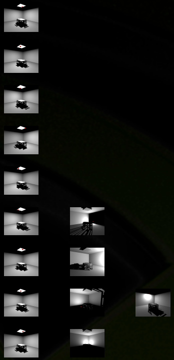

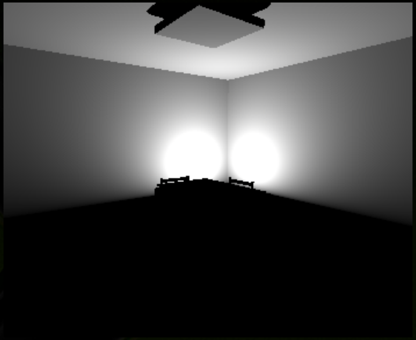

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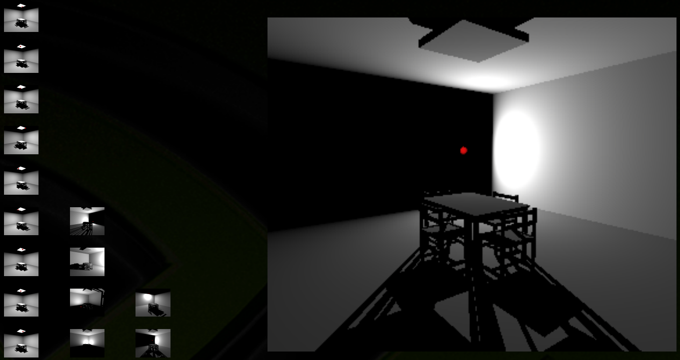

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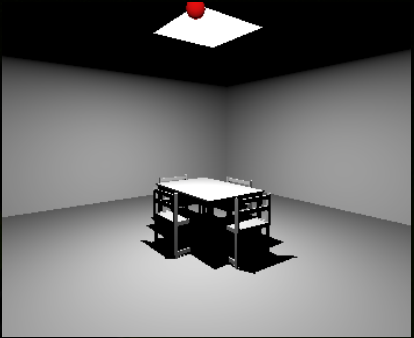

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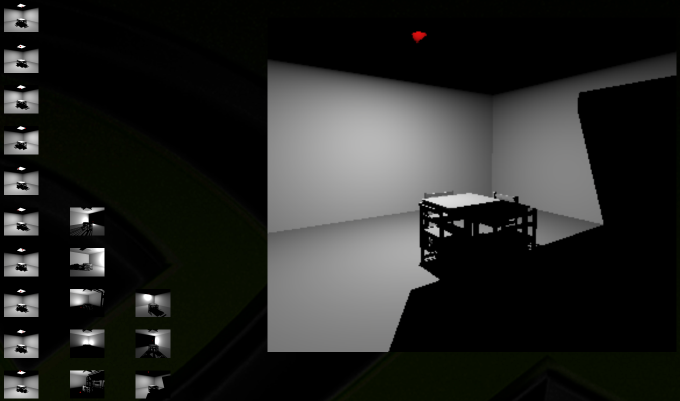

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Path Space Partitioning

The weak singularity

- at that time, hardware avoided overmodulation of geometric term

$$G(x_j, y) := V(x_j, y) \frac{\cos\theta_{x_j} \cos\theta_y}{|x_j - y|^2}$$

for small distances $|x_j - y|^2$ and visibility $V(x_j, y) = 1$ by clipping

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- computing the missing contribution in a robust way

$$L_d(y, z) \quad = \quad \int_{\operatorname{supp} L_e} L_e(x, y) f_r(x, y, z) G(x, y)\, dx$$

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$$\begin{aligned} L_d(y,z) &= \int_{\operatorname{supp} L_e} L_e(x,y) f_r(x,y,z) G(x,y)\, dx \\ &\approx \int_{\operatorname{supp} L_e} L_e(x,y) f_r(x,y,z) \min\{G(x,y), b\}\, dx \end{aligned}$$

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Path Space Partitioning

The weak singularity

- algorithm

$$L_d(y,z) \approx \frac{|\operatorname{supp} L_e|}{n} \sum_{j=0}^{n-1} L_e(x_j,y) f_r(x_j,y,z) \min\{G(x_j,y),b\}$$

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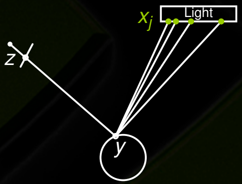

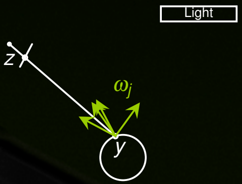

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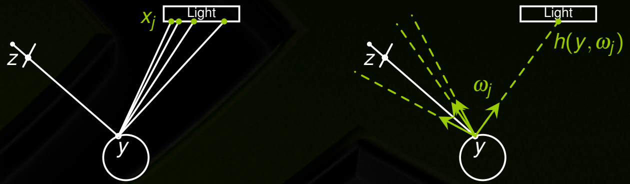

Path Space Partitioning

The weak singularity

- biased result of only bounding the weak singularity

Path Space Partitioning

The weak singularity

- consistent simulation

Consistent Generation of Light Transport Paths

Monte Carlo and quasi-Monte Carlo integration

- consistency matters most

$$\mathbb{P}\left(\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n-1}f(x_i)-\int_{[0,1)^s}f(x)dx=0\right)=1$$

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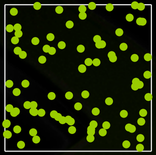
  - example: sampling an area light source



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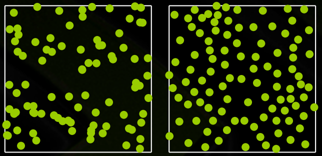

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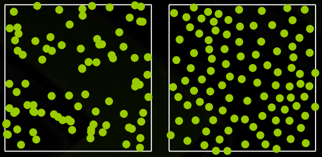

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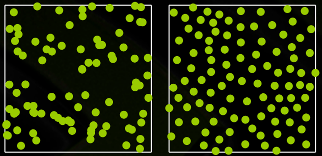

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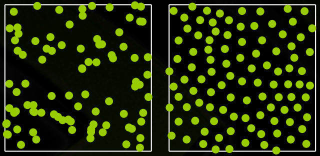

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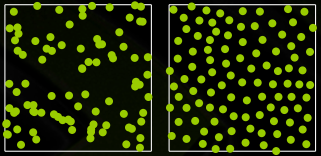

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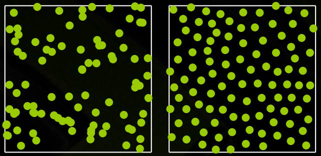

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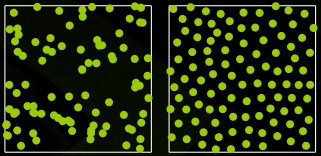

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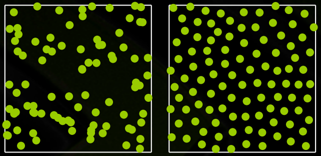

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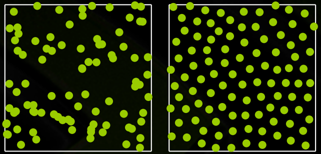

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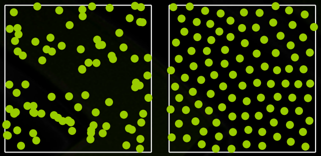

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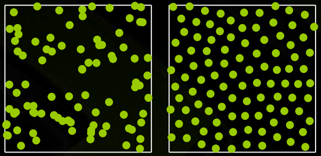

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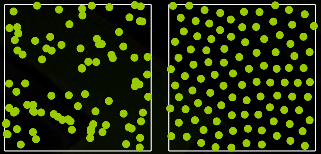

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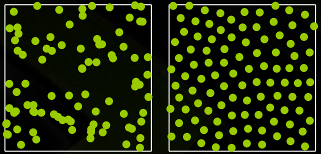

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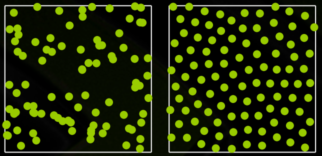

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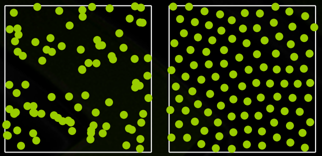

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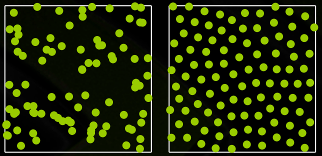

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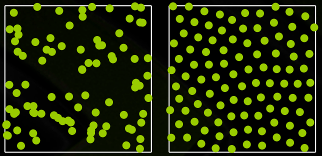

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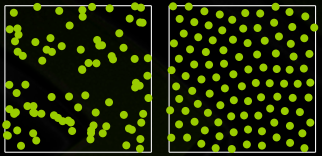

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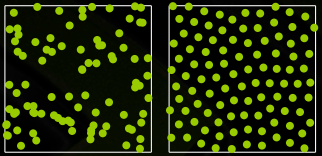

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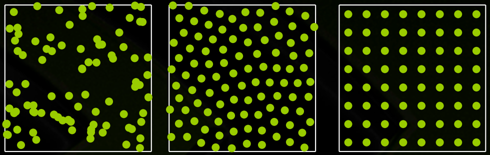

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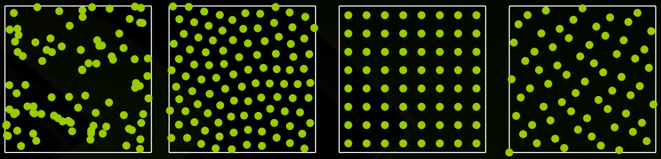

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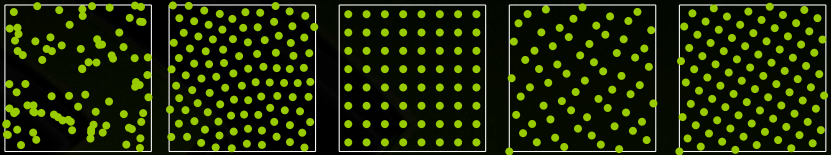

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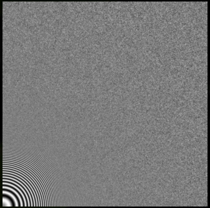
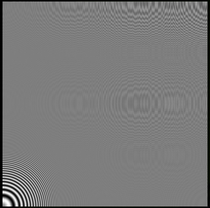
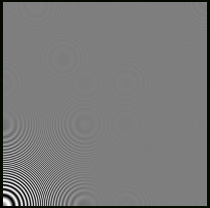
  - example: anti-aliasing a zone plate at 4 samples per pixel
    - aliasing independent of numerical integration scheme



jittered sampling



(t,s)-sequence



rank-1 lattice

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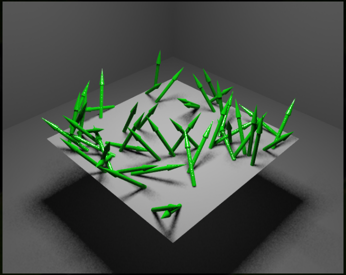
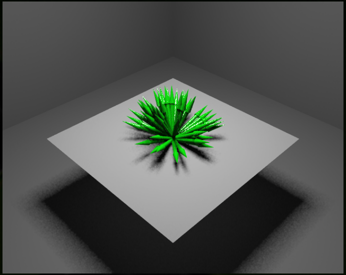
  – example: emission and scattering using pseudo-random sampling





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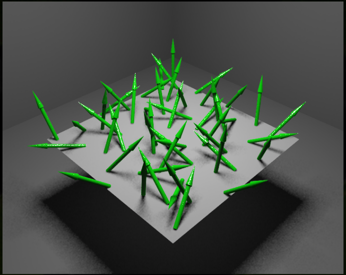
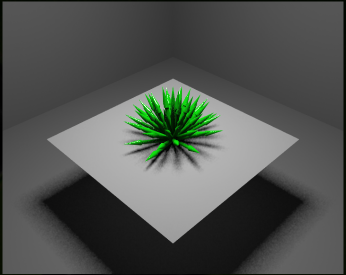
  - example: emission and scattering using low discrepancy sequences





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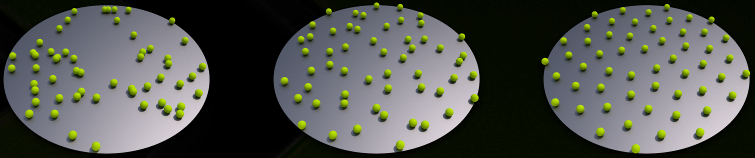

random   Halton sequence   Fibonacci lattice

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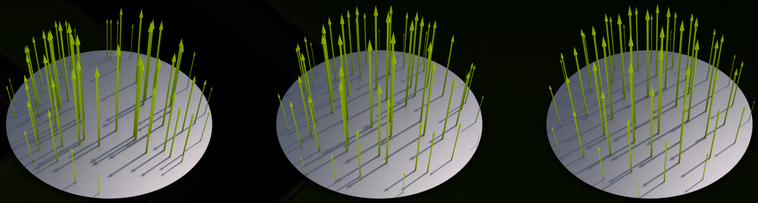

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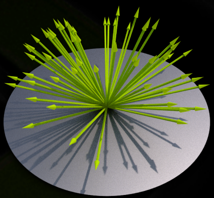

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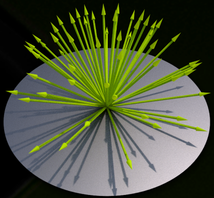
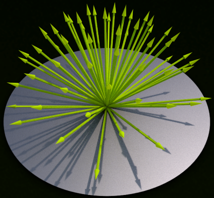


Halton sequence



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You still don't trust deterministic consistent sampling ?

- come to the course

Advanced (Quasi-) Monte Carlo Methods for Image Synthesis

Thursday, 9 August 9:00 AM - 12:15 PM — LACC - Room 406AB

15 Years of Instant Radiosity

Hindsights

- improved accumulation buffer
  - added indirect illumination by using many point lights
    - unlucky formulation of ”operator norm estimation”
  - introduced quasi-Monte Carlo methods for light transport simulation
    - much more advanced by today
- limitations of rasterization
  - generating the light paths using the rasterizer is tricky
  - reflection and refraction are not feasible in a general way

Acknowledgements

- Peter Schröder